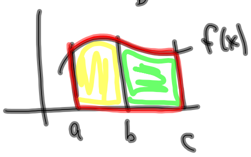
5.3b Definite Integrals and Antiderivatives

Rules for Definite Integrals

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \underline{\int_a^c f(x)\,dx}$$



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$$\int_1^3 f(x)\,dx = 5 \qquad \int_1^7 f(x)\,dx = -10$$

$$\int_3^7 f(x)\,dx = ?$$

$$\int_1^3 f(x)\,dx + \int_3^7 f(x)\,dx = \int_1^7 f(x)\,dx$$

$$5 + \int_3^7 f(x)\,dx = -10$$

$$\int_3^7 f(x)\,dx = -15$$

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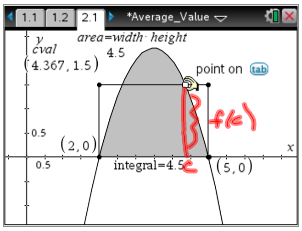
Average (Mean) Value



$$f(c)(b-a) = \int_a^b f(x)\,dx$$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

average value of f(x) on [a,b]

$$\boxed{\bar{y} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

average value

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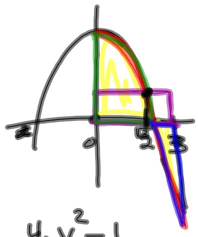
Find the average value of f(x) = 4-x^2 on [0,3]. Does *f* actually take on this value at some point on the given interval?

$y = 4 - x^2$



$$\bar{y} = \frac{1}{3-0}\int_0^3 4 - x^2\,dx$$

$$= \frac{1}{3}\left[4x - \frac{x^3}{3}\right]_0^3$$

$$= \frac{1}{3}\left[4\cdot 3 - \frac{3^3}{3} - 0\right]$$

$$\bar{y} = \frac{1}{3}[12 - 9] = 1$$

$4 - x^2 = 1$

$3 = x^2$

$x = \sqrt{3}$ yes

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Mean Value Theorem for Definite Integrals

If $f(x)$ is continuous on $[a,b]$

then there exists a c between a & b so that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f(c) = \bar{y}$$

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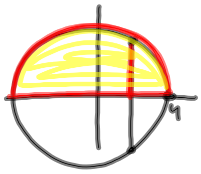
How long is the average chord of a circle of radius 4? Find the value that satisfies the Mean Value Theorem for Definite Integrals.



$$y = \sqrt{4^2 - x^2}$$

$$\bar{y} = \frac{1}{4--4}\int_{-4}^{4}\sqrt{4^2-x^2}\,dx$$

$$= \frac{1}{8}\int_{-4}^{4}\sqrt{4^2-x^2}\,dx$$

$$\bar{y} = \frac{1}{8}\cdot\frac{1}{2}\pi\cdot 4^2 = \pi$$

$$\sqrt{4^2-x^2} = \pi$$

$$4^2 - x^2 = \pi^2$$

$$x = \sqrt{4^2-\pi^2} = \boxed{2.47596 = c}$$

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